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A FEW CONSTRUCTIVE PHASES OF MATHEMATICS IN LIFE

By W. PAUL WEBBER, J. P. COLE and R. L. O'QUIN
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In this age no country can compete successfully in business or industry without advanced and specialized scholarship in many fields of study. So well recognized is this that we read in current newspapers that more and more research fellowships are being endowed in our great universities by commercial and industrial corporations. Fire insurance companies are providing facilities for young people to study fire insurance scientifically. Fellowships in agriculture, economics and various other lines are being added to the list. Even fellowships in handling and marketing meat are to be found.

It will be noticed that a number of these fellowships are more or less vocational in character, but they are not to be confused with the "shop hand" or "manual arts" type of vocations. They require intellectual application of a high order. The fellows are to carry on studies of an advanced and quantitative nature in their respective fields. A number of "academic" subjects may be necessary as a preparation for the work. It is known that the fellows may fail to obtain satisfactory results unless their preparation is adequate to the proposed study.

Among the foundation studies for eligibility to these fellowships will be found Physics, Chemistry, Psychology, Statistics and Mathematical studies of various kinds, economics, higher accounting, etc. To say that the necessary knowledge of these may be picked up as the fellow proceeds with his investigations is to propose too great a handicap on him to allow him a fair chance of success in average cases. Often the need for investigation comes suddenly and results must be obtained in a short time. Some examples below will make this point clear.

Most of the foundation subjects named above are available and useful for research purposes only in more or less mathematical form. Considerable mathematical knowledge will be necessary to read the literature of these subjects for research purposes.

Many years ago, Nicholas and Franklin said in the preface to their text on physics, "Calculus is the natural language of physics." Today the same may be said of other subjects, if we substitute "mathematics" for "calculus." Of course calculus is only one of a number of advanced mathematical methods. In fact all science tends to become mathematical. This has been expressed by another as follows—"The finished form of all science is mathematical."

As civilization (What ever that means) progresses, every endeavor of man is subjected to scientific study. Even man's pastimes and frivolous efforts are not excepted. For history tells us that a scientific study of gambling, as a game of chance, has lead to the very abstruse and useful doctrine of chances or probability is fundamental in the theory of education, for it is at the very foundation of all statistical studies. Now we read that collegiate athletics is to be subjected to a scientific investigation. This can only mean that in part the study must be mathematical. What a fate for the game enjoyed by many students who did love mathematics. Surely time is cruel. We shall show below that national defense has depended for its success upon the power of the expert mathematician to analyze and interpret the laws of nature for the war department and that none but the mathematician could have done it.

In a democracy it is assumed to be fundamental that the State should furnish opportunity for so much education to its citizens as may be necessary for the success, safety and happiness of the people as a state in an association of competing states. This means the state must guarantee to a sufficient number of its capable young people so much education in various lines as may be necessary to its maintenance, welfare and advancement among other states.

It is the purpose of this paper to point out a few actual examples of the many activities in which mathematics functions in the great scheme of human life. It will be seen from these examples that any scheme of higher education that neglects to offer a fair modicum of mathematical training falls short of preparing for the foundations subjects which are the basis of all applications of scientific study and fails to assure to the state a sufficient

number of masters in these various fundamental studies on which the welfare and safety of the state depend.

Example 1. In pre-war days an employee of a large bridge building corporation came to one of the member of this staff with a statement about as follows: "I have reached my limit unless I get more mathematics. I now make \$5,000 a year but I can go higher if I learn trigonometry. Will you teach me trigonometry and what will it cost me?" Here was a \$5,000 man up against it and asking a \$1,500 college instructor to help him to prepare to make \$6,000 or \$7,000.

Example 2. Experimental methods in educational theory developed rapidly in recent years. There was a strong demand for statistical calculations. A number of primers of statistics came off the press in a short time. Students and teachers of education used these books in evaluating their data. Often there was lack of knowledge of the nature of the formulas used and what should have been valuable conclusions were later found to be unreliable. Here was a direct loss to the state. It seems but fair to suggest that expert educational experimenters should take into confidence some capable mathematician. This can be done by a little co-operation.

Example 3. The Federal income tax law made a sudden and large demand for certain types of higher accounting and statistical methods. Often men not realizing the nature of the work assumed to undertake it. As a result many errors were made and much loss was incurred both to business and to the government. Later these errors had to be eliminated by slow and expensive work.

Example 4. The physical sciences all originated in qualitative laws derived from first observations. Later these were corrected by quantitative observations and reduced to exact laws quantitative in character. To illustrate: Considerable was known in a qualitative way about electricity for a long time before we had electric railways. It was not until the knowledge of electricity was put in mathematical form that our modern electrical conveniences became possible on a commercial scale.

Example 5. Mathematics has been especially necessary to the successful competition of this country in military affairs. The

European war brought this out very distinctly. The nations that had skilled mathematicians in their ordnance departments were the hardest to beat. It was not a struggle of arms alone, but a struggle of mathematics. Since bullet firing weapons came into use the nations have been experimenting to secure some advantage. Previous to the European war little had been done since 1870 to increase the range of artillery. During the war there arose the sudden need of competing with an enemy nation that had already applied mathematics to the subject. The problem was complicated and only the most highly trained men could cope with it. The various forces that had to be considered were well known individually but in long range firing a number of such forces had to be considered simultaneously. If some of the professors of mathematics of one of our great universities had not volunteered into the ordnance department the war would have been greatly prolonged, if not lost. These men with mathematics alone applied to the laws of nature were able in a short time to put our batteries on an equality with any in the enemy lines. They were even superior in their class of artillery. The war department was unable, without this aid, to solve the problem. Since the war another mathematician from another great university has, by an application of higher mathematics, trebled the efficiency of the small arms. He was able to determine the correct form of projectile for the greatest range and accuracy. Not only was the range trebled but the accuracy at the increased range was made better than it had been at the old range. Experiment and empirical methods had reached a stopping place that might have taken many years to get away from. In a single stroke mathematics made the improvement. What has happened may happen again in a different form. Surely national defense is a state affair. Surely state institutions are justified in supporting departments where men of such usefulness, as above indicated, can be trained and held in readiness for an emergency.

Example 6. Much has been done to teach us that discipline in education is deceased. Many educators profess to believe that it is now lying peacefully in its grave. Be that as it may, there are a number of practical people who still obtain aid from the idea of discipline. Mathematics has something to offer to the man who is preparing for a vocation which does not appear to have

any particular relation to mathematics. If an exercise of logic is required, the habit of thinking logically from cause to effect and following an unbroken chain of logical deductions can be successfully instilled by a suitable course of mathematical training. The generality of mathematical methods will constitute an advantage over most other studies. Many lawyers cultivate mathematics for this purpose. A point worth noting here was made in a well thought out address at the meeting of the mathematics section of the State Teachers Association. The speaker said that many great men in various fields of life are also mathematicians of considerable skill. Examples of such men were cited, where it was shown that achievements in various fields were aided by the study of mathematics. But it was not generally known that these men were students of mathematics.

The applications of mathematics to industrial problems and to national defense are by no means the only important uses of mathematics in civilization. We make the assertion that social, political and religious intolerance have their anchorage in lack of knowledge of the laws of nature, (Psychology is here included among the natural sciences.) Many people have no adequate idea of the vastness, beauty, and law abiding character of nature as exemplified in this universe. The writers have daily evidence of these conditions in their work. It is one of their duties to unshackle the minds of students from the restricted vision of childhood and isolation. One of the finest studies for this is astronomy. The vast distances dealt with and the surprising accuracy of the astronomers calculations and his predictions of eclipses are matters for thought. They are illuminating to the mind. Astronomy is a mathematical science. Many of its interesting facts can be grasped with a relatively small knowledge of *difficult* mathematical processes. We believe that a study like astronomy broadens the mind and make for high ideals and for tolerance in the minds of those who study it.

We cite two examples of the use of advanced mathematical methods in astronomy. Many years ago before the planet Neptune was discovered it was found that the planet Uranus was not coming round the sun on schedule time, the difference was too small to be noticed without a telescope. It was sufficient, however, for the basis of a mathematical calculation made independ-

ently by two men who arrived at substantially the same results, Viz: The existence of a hitherto unknown planet beyond Uranus. This stranger was pulling Uranus out of her course. Within an hour after the calculations were sent to an observatory the stranger was discovered and within twice the width of the moons disc from the predicted place. The motion of Neptune is now being studied carefully with the view to determining the presence of another, the ninth, major planet in our system. It is noted that these bodies are thousands of times farther from the sun than is the earth.

In recent years small discrepancies were discovered in certain physical laws. Physicists were much puzzled and could only explain the difficulties by making their laws more complicated, whereas they believed that nature operated by simple laws. Finally an explanation came in purely mathematical form. A mathematician by the name of Einstein was able to simplify the difficulty and enable science to go on with its work. This discovery is far reaching not only in its application to physics but it illuminates the mind in a far more general way by pointing out an attitude hitherto not attainable owing to our restricted vision. This discovery will make for breadth and tolerance as it becomes more generally understood.

Many centuries ago, in ancient times, there was a theory of the motion of the planets and stars that had gained approval in official circles. The discovery of increased variations of motion unknown to the previous astronomers became a burden to the theory and rendered their explanation well nigh impossible by any but the most fantastic suppositions, and complicated set of curves must, by the old theory, be circles. Why? Because the circle was the only perfect curve and God would not make heavenly bodies move in any but perfect curves. Other curves have since been discovered and it has been shown that the planets do not move in circles. To the old theory in its restricted view none but circles could be perfect. The study of mathematics applied to astronomy have shown beyond doubt that there are many kinds of curves, not circles, that to all appearance are just as perfect and just as beautiful as circles.

We assert on the basis of these examples that the study of mathematics enables one to understand nature better and as a

consequence it unshackles the mind and makes for the going forward of civilization in a more intelligent way. If old pet theories of science or of philosophy must be laid aside, so be. We can and should reverse our forebears for what they did but that should not blind our vision to new and better things that we can discover and leave to our children. Mathematics will assist in opening the secrets of the universe to all who will earnestly study it. So true is this that we feel inclined to make the assertion that if one has a good knowledge of two things he has the key to all human knowledge, viz.—Learn English as a means of securing and communicating knowledge. Learn mathematics as a means of studying all natural laws. If one could have school instruction on but two subjects, should they be mathematics and English?

The engineering profession is recognizing the decline in available natural resources together with the decline of sporadic inventions. "There is a narrowing margin between invention and scientific research." "We must look in the research laboratory for our inventions in the future." The days in which great inventions come by accident and in isolation are past. Invention must follow research as a conscious need in civilization, continue as an accidental luxury. This can only mean that, eventually, under conditions such as we can picture, industry will take pure scientific research "right from the bat in the field" and not in the far out field. Scientific research must depend in no small measure on mathematics. Hence it is a duty to make ample provision, in all scientific education and in the preparation of high school teachers, for a sufficient basic training in mathematics.

TEACHING LOGARITHMS TO HIGH SCHOOL PUPILS IN EIGHT RECITATION PERIODS

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For the past five years it has been the lot of the writer as a teacher of Algebra to teach "Logarithms" twice a year, and twice a year he resumed his search for improved methods that would insure thoroughness both in the theory and the technique of logarithmic computation without exceeding the time limit set to the subject. Out of this periodic experimentation a plan was evolved that proved more effective than any the writer had previously tried. Following is a detailed outline of the subject as taught complete in eight recitations:

RECITATION I—MEANING OF "LOGARITHMS" AND THEIR USEFULNESS

The first lesson begins with a rapid review of the laws of exponents, where particular emphasis is laid on numerical bases and the attempt is again made to open the more reluctant minds to the truth that $10^0 = 1$ (not 0), that $3^2 \times 3^3 = 3^5$ (not 9^{10}), and that $\sqrt{9^4} = 9^2$ (not 3^2). In the meantime a pupil who can be "spared" from this review is directed to work out on the board a table of the first 20 consecutive powers of 2 thus:

$$\begin{aligned}2^1 &= 2 \\2^2 &= 4 \\2^3 &= 8 \\2^4 &= 16\end{aligned}$$

To this table the attention of the pupils is called as soon as they show sufficient skill in applying the exponent laws. A challenge to the class to discover an easy way of multiplying two numbers from the table, like 512 and 1024, stirs up a good deal of mental activity. As a rule the discovery is made. In less responsive classes the reaction may be hastened by a leading

question as: "what two expressions may be multiplied instead of 512 and 1024." The equation of $2^9 \times 2^{10} = 2^{19}$ is produced and the answer 524288 is read out of the table. When it thus dawns on the pupils that they can actually find at a glance the product of 512 and 1024, or the square root of 1048576, or the cube root of 262144 their satisfaction is immense. They delight in the drill that follows—it is indeed so easy!

$$\sqrt[3]{32768} = \sqrt[3]{2^{15}} = 2^5 = 32$$

$$262144 \div 4096 = 2^{18} \div 2^{12} = 2^6 = 64$$

$$\sqrt{1048576} = \sqrt{2^{20}} = 2^{10} = 1024$$

There is no need for arguing that multiplying "by exponents" saves time and labor. The class is already convinced.

Here the term "logarithm" may be introduced. That there is in the equation $2^5 = 32$ a close relationship between 5 and 32 is only too apparent. 5 is something of 32—let this "something" be called "logarithm." The logarithm of 32 then is 5 provided, of course, 2 is the base and the logarithm of 1024 is 10; and so the logarithms of the other numbers in the table are recognized. Then more drill—questions of the type: "What is the logarithm of 64?" Somebody in the class will suggest that "it all depends on the base," and soon the truth crystallizes that 64 has 2 for a logarithm if 8 is the base, 3 if the base is 4 and 6 if the base is 2. Thus gradually, slowly, and, for the most part, painlessly the pupils begin to "feel" the meaning of "logarithm"—a feeling which, if properly cultivated, will ripen into complete understanding.

For outside work the pupils are asked to develop a table of the first 20 quarter powers of 10 from the three initial values.

$$10^{0.25} = 1.7782$$

$$10^{0.50} = 3.1623$$

$$10^{0.75} = 5.6234$$

The assignment also includes a set of examples on changing exponential equations to the logarithmic form and vice versa.

RECITATION II—FINDING COMMON LOGARITHMS OF NUMBERS

At the beginning of the second recitation the class is sent through a rapid drill on the subject matter of the previous lesson, the table of the quarter powers of 10 now supplying the situations for the various questions. More and more the pupils come to see the meaning of logarithms and their effectiveness in simplifying multiplication, division, involution and evolution. Some inquisitive member of the class will then ask (or be led to ask), "what about the numbers that are not exact powers of 2 or those not represented on the table of the powers of 10?", and thus unwittingly suggest the text for the new lesson. The class is informed that benevolent mathematicians of the past had dedicated their lives to the project of expressing ALL numbers as powers of 10 and gave to the world the results of their labors as a permanent gift, a heritage to mankind. There is at present, the pupils are told, a simplified and perfected table of common logarithms available for their use if they but learn how to use it. The class now addresses itself to the task of learning how to find logarithms of numbers with a curiosity and a willingness unparalleled in other Algebra work.

That most logarithms are made up of two parts is easily seen from this table suggested in all text books on Intermediate Algebra.

$10^0 = 1$	$\log 1 = 0$
$10^1 = 10$	$\log 10 = 1$
$10^2 = 100$	$\log 100 = 2$
$10^3 = 1000$	$\log 1000 = 3$

It takes but a minute for the pupils to realize that numbers which are not exact integral powers of 10 will not have exact integral logarithms, that 376, or 594, or 104 being situated between 100 and 1000 will have a logarithm larger than 2 and less than 3, or 2 and a fraction; and similarly for other numbers, larger than 1, that are not exact powers of 10. There seems to be at this point a need for naming the two parts of a logarithm, and the pupils are satisfied with "characteristic" for the integral parts and "mantissa" for the fractional part. The comparison of a few numbers with the characteristics of their logarithms leads the class to discover the rule that "the characteristic of the

logarithm of a number greater than 1 is one less than the number of significant figures to the left of the decimal point."

The rule for numbers less than 1 calls for more explanation and more drill. From the table

$10^0 = 1$	$\log 1 = 0$
$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^{-2} = 0.01$	$\log 0.01 = -2$
$10^{-3} = 0.001$	$\log 0.001 = -3$
$10^{-4} = 0.0001$	$\log 0.0001 = -4$

it is evident that a fraction like .037 which is larger than .01 and less than .1 will have a logarithm larger than -2 and less than -1 or $-2 +$ a positive fraction. Experience has taught the writer that he cannot lay too much emphasis on the point that a number less than 1 has a logarithm with a negative characteristic. Further study of the above table supplemented by the following brings out the unique relationship between the number

	of zeros preceding the
characteristic of $\log 0.78 = -1$	first significant figure
characteristic of $\log 0.078 = -2$	and the numerical value
characteristic of $\log 0.0078 = -3$	of the characteristic.
characteristic of $\log 0.00078 = -4$	Again the class is aided
characteristic of $\log 0.000078 = -5$	in formulating the rule

that "the characteristic of the logarithm of a number less than 1 is numerically one more than the number of zeros between the decimal point and the first significant figure." The pupils are drilled for a short time on the application of the two rules for finding characteristics and are constantly urged to keep the derivation of the rules clearly in mind so as to be able to reason themselves to safety in case of any doubt.

Mantissas for numbers of not more than three significant figures offer no particular difficulty. The following statements from the table of the powers of 10 are placed before the class.

$10^{0.25} = 1.7782$	$\log. 1.7782 = \underline{0.25}$
$10^{1.25} = 17.782$	$\log. 17.782 = \underline{1.25}$
$10^{2.25} = 177.82$	$\log. 177.82 = \underline{2.25}$
$10^{3.25} = 1778.2$	$\log. 1778.2 = \underline{3.25}$

The pupils are asked to explain why the logarithms of 1.7782, 17.782, 177.82, and 1778.2 have the same mantissa. It materializes eventually that mantissas are alike in logarithms of numbers composed of the same significant figures arranged in the same order—that is mantissas are independent of the positions of decimal points. Thus numbers like 2, 20, 2000 and .02 must have the same mantissa. The pupils are then directed to the four-place table in their text-books and shown how to find the mantissa for a number of not more than three significant figures. An interesting correlation with Geometry is possible here. Classes in "Logarithms" are pleased to recognize in the process of finding the mantissa for a number like 573 the solution of a compound locus problem, as the required mantissa is the "intersection" between the horizontal line of mantissas headed by 57 and the vertical line headed by 3.

Through the remaining minutes of the period the class is drilled on writing logarithms complete for numbers of not more than three significant figures. Negative characteristics are written in both forms:

$$\log .031 = 8.4914 - 10$$

$$\log .031 = 2.4914 \text{ (apologies for this somewhat antiquated form to be offered later.)}$$

RECITATION 3—INTERPOLATION

This lesson is devoted to interpolation. And here, as the class is facing the less spectacular features of "Logarithms," the teacher is threatened with a decline in interest among his pupils and a corresponding lessening in effort. It is a crisis for which he must be prepared. Interpolation may seem easy to a teacher but it is difficult for the beginner as it calls for a kind of reasoning to which the high school junior is not accustomed. To allow for the needed readjustment, the approach must be slow, gradual, every step thoroughly explained—interpolation, in fact, cannot be overexplained.

Let it be required to find $\log 59.43$. It is made clear at the outset that the characteristic takes care of the decimal point,

and so far as the mantissa is concerned the decimal point ceases to exist. The mantissa for 5943, then, is to be found, one that cannot be read out directly from the table. The class is asked to suggest the two numbers nearest to 5943 that do have mantissas in the table. After a few wild guesses the right numbers 5940 and 5950 are named. The mantissas of these numbers are determined and placed on the board as follows:

Mantissa for 5950	= .7745
Mantissa for 5940	= .7738
Difference between numbers	= 10
Difference between mantissas	= .0007

First the pupils are led to see that, because 5943 is between 5950 and 5940, the mantissa for 5943 will be between .7745 and .7738, that is less than the first and greater than the second, or mantissa for 5943 = $.7738 +$ a certain difference. It is a good plan to pause here for a minute and let the less permeable minds take in what has thus far been offered. The next thing that the pupils must be made to see and thoroughly understand is that the difference to be added to .7738 is less than .0007, the total tabular difference. Only a part of .0007 is to be added, what part then? The whole of .0007 is added to .7738 for a number that is by 10 larger than 5940. Assuming that the increase in the mantissa is proportional to the increase in the number, it seems that .1 of .0007 must be added for a number that is by 1 larger than 5940, and .2 of .0007 for a number that is by 2 larger than 5940, and .3 of .0007 for a number that is by 3 larger than 5940. Hence the mantissa for 5943 = $.7738 + .3 (.0007) = .7738 + .0002 = .7740$; and $\log 59.43 = 1.7740$. Here again the class is allowed time to think it over and ask questions.

A comparison of the problem just discussed with the case where the number is made up of five significant figures shows that very little difference exists between the two problems. The number 59439 has for its nearest representatives on the table of mantissas the numbers 59400 and 59500 with mantissas .7738 and .7745 respectively. Hence by reasoning similar to that used above it follows that mantissa for 59439 = $.7738 + .39 (.0007) = .7738 + .0003 = .7741$. The rest of the period is

given over to drill where each member of the class has a chance to show his ability to interpolate or the lack of it, the latter cases receiving the necessary individual attention. The assignment is a set of examples on finding logarithms of numbers no restriction now being made on the number of significant figures. The following form, however, used throughout the drill, is rigidly insisted upon for the outside work:

Problem—Find $\log .0079276$

Solution

Mantissa for 79300 = .8993

Mantissa for 79200 = .8987

100 .0006

Mantissa for 79276 = .8987 + .76 (.0006) = .8987 + .0005 = .8992

$\therefore \log .0079276 = \bar{3}.8992$

RECITATION 4—OPERATIONS ON LOGARITHMS

Having learned how to find logarithms of numbers, the class is now ready to learn how to use them, when found, towards the solution of various problems. From the table of the powers of 2 a few statements are produced on the board. A pupil is asked

$2^6=64$	to give the product of 256 and 2048. The
$2^7=128$	analysis of the process that yields 524288 re-
$2^8=256$	veals the fact that the product is secured
	through the number 19 which is easily recog-
$2^{11}=2048$	nized as the logarithm of 524288 (2 being the
	base.) It is also evident that 19, the key to the
$2^{18}=262144$	answer, is a result of addition not multiplica-
$2^{19}=524288$	tion—addition of 8 and 11 which are the
$2^{20}=1048576$	logarithms of 256 and 2048 respectively. Hence
	the conclusion is warranted that "the logarithm
	of a product is equal to the sum of the loga-
	rithms of its factors." Likewise an example on division leads to
	the formulation of the rule that "the logarithm of a quotient is
	equal to the logarithm of the dividend minus the logarithm of

the divisor." A few minutes of appropriate drill in connection with common logarithms will help pupils form the habit of applying the rule correctly.

Next the class proceeds to modify the exponent laws for powers and roots to fit the needs of logarithms. From the process of involution $16^3 = (2^4)^3 = 2^{4 \times 3} = 2^{12} = 4096$ it is clear that the key number "12," the logarithm of the answer is the result of multiplying 3, the exponent of the power, by 4, the logarithm of 16. The rule is now stated in the general form that "the logarithm of any power of a number is equal to the logarithm of the number multiplied by the exponent of the power." And through the evolution process $\sqrt[3]{262144} = \sqrt[3]{2^{18}} = 2^{\frac{1}{3} \times 18} = 2^6 = 64$ the rule is derived that "the logarithm of the root of a number is equal to the logarithm of the number divided by the index of the root." Then more drill to fix these principles in the minds of the pupils. The formal generalized proofs for the properties of logarithms are excluded from this discussion since, in the opinion of the writer, the contributions made by these abstract demonstrations to the pupils' understanding of logarithms are not sufficient to warrant the time and energy spent on them.

A set of examples on finding logarithms of products, quotients, powers and roots is assigned for the coming recitation and the remainder of the period is devoted to "supervised work" on the assignment. In the course of this supervision great stress is laid on the form in which the solution of a problem is to appear—as many failures on logarithmic problems are directly traceable to lack of system in the methods of solution. The writer, therefore, insists that all the computation involved in solving a problem by logarithms, including interpolation and other minor operations, be carried out systematically on one and the same paper. No defence for this strict requirement need be offered to Algebra teachers undoubtedly familiar with the type of a paper exhibiting a wrong answer amid a confusing display of unnamed logs, antilogs and tabular differences with the sources of the trouble lost somewhere on the back of an envelope, desk cover or even shirt sleeves. Following is an extract from a pupil's paper showing the form required for problems on the present subject matter:

Problem—Find $\log 4\pi r^2$, if $r = 2.0667$

Solution

$$\begin{aligned}\log 4\pi r^2 &= \log 4 + \log 3.1416 + 2 \log 2.0667 \\ \log 4 &= 0.6021 \\ \log 3.1416 &= 0.4971 \\ 2 \log 2.0667 &= 0.6306 \\ \log 4^2\pi &= 1.7298\end{aligned}$$

Man. 31500 = .4983	16
Man. 31400 = .4969	14
<u>100</u> <u>.0014</u>	<u>64</u>
	16
	<u>224</u>
Man. 3.1416 = .4969 + .0002 = .4971	

Man. 20700 = .3160	21
Man. 20600 = .3139	67
<u>100</u> <u>.0021</u>	<u>147</u>
	126
	<u>1407</u>
Man. 20667 = .3139 + .0014 = .3153	
2 log 2.0667 = 2 × 0.3153 = 0.6306	

RECITATION 5—FINDING THE NUMBER CORRESPONDING
TO A LOGARITHM

But one more link is now needed to complete the chain. The pupil is yet to learn how to reclaim numbers from their logarithms, and the teacher is yet to teach his hardest lesson on logarithms. For the operation of finding the antilog of a logarithm must be understood in all its phases if it is to be performed successfully. Now and then a pupil may surprise his teacher by showing himself able to find logarithms of numbers correctly though ignorant of the theory involved, but he is very unlikely to duplicate the feat when the direction of the process is reversed. Here pupils must be taught to reason—something to be achieved only through a very careful approach and gradual climbing.

It is required, for example, to find the antilog of 3.7059. First the fact is driven home to the pupils that the number sought bears no physical resemblance to 3.7059, just as there is no resemblance between 20 and its logarithm 1.3010. A brief inquiry into the meaning of the characteristic "3" makes it clear

that the required number is greater than 1000 and that there will be in the number 4 figures before the decimal point. Further drill on this phase of the work is most timely. What if the characteristic were 2?; 1?; 0?;—2? The habit of thus "sizing up" the number from its logarithm is very effective as a means of preventing illogical conclusions, and is encouraged throughout the remaining lessons on logarithms. The class is now offered the following suggestion for approval: to be able to write the number whose logarithm is 3.7059 one must know, (1) the figures of which the number is composed, (2) the exact position of the decimal point. The suggestion found reasonable, pupils are led to recall that the line is indeed sharply drawn between the parts of a logarithm, the characteristic having nothing to do with the composition of the number and the mantissa being equally free from any connection with the decimal point. The characteristic then can be relied upon to give the position of the decimal point, while the mantissa must be explored for the figures of which the number is composed. The pupils turn to the table of mantissas and find the given mantissa (.7059) on the horizontal line headed by 50 and the vertical line headed by 8. The number, now known to be made up of the figures 5, 0 and 8, is still of uncertain value as it may be 508, 50.8, .00508 and what not. The characteristic, however, and the advance estimate lead the pupil to expect a number larger than 1000 and less than 10000, hence the answer is fixed at 5080 (approximately).

The solution of a problem involving interpolation is undertaken next.

$$\log x = \bar{2}.7188 \qquad x = ?$$

The number is at once associated with a fraction greater than .01 and less than .1 and the answer is visualized as a decimal fraction with one zero inserted between the decimal point and the first significant figure. Incidentally, for purposes of finding antilogs the form $\bar{2}.7188$ is unquestionably preferable to the form 8.7188-10. The given mantissa is not found in the table. The two mantissas nearest to .7188 are recorded with their numbers thus.

.7193=M 524	.7188
.7185=M 523	.7185
<u>.0008</u>	<u>.0003</u>
1	

The number (disregarding the decimal point) corresponding to .7188 must be between 523 and 524, or $523 + \text{a fraction}$. The size of this fraction, dependant apparently on the difference between the given mantissa and that of 523, is reasoned out as follows: a number (524) that is by 1 larger than 523 has a mantissa by .0008 larger than the mantissa of 523; it seems logical then that a number (x) whose mantissa is by .0003 larger than the mantissa of 523 will be by .0003 of 1 or $\frac{3}{8}$ of 1 larger

$$\frac{.0003}{.0008}$$

than 523. Hence the required number is $523\frac{3}{8}$ (disregarding the decimal point). The fraction is converted to the decimal system, the characteristic consulted and the complete answer written $x=.0523375$. The attention of the class is here called to the fact that the above answer is accurate arithmetically but practically, since in a four-place table the last figure in any mantissa is only an approximation, the accuracy of the last two figures can be seriously questioned.

A few problems to be solved completely by logarithms are assigned and the work on the assignment begins at once under supervision.

RECITATION 6—RECAPITULATION AND INDIVIDUAL INSTRUCTION

This period begins with a short written test carefully designed to reveal the weaknesses of individual pupils. When the returns on this test are all in, there is ample indication as to where individual help is needed. The class is started at once on the new assignment, a set of problems differing from those assigned the day before only in the degree of difficulty. The work is closely supervised, with pupils that "had made good" pressed into service as assistants. Particular attention is given those that had failed on the test, the trouble in each case being traced to its source and a remedy suggested. Difficulties that appear to be more or less general are taken up with the whole class. Thus some pupils may be found weak in performing operations on logarithms with negative characteristics. The matter is then discussed in detail.

In the problem that follows the sum of four logarithms is to be determined. It is evident that the sum of the four positive

$\begin{array}{r} \bar{2}.5792 \\ \bar{3}.4921 \\ 0.6357 \\ 3.1629 \\ \hline 1.8699 \end{array}$	<p>mantissas is $+1.8699$. Of this the fraction .8699 is the mantissa of the answer, while the $+1$ becomes a characteristic and is added to the characteristics -2, -3, 0 and 3 to give a sum of -1. Hence the answer complete is $\bar{1}.8699$. The writing of the minus signs over the characteristics in the first two logarithms seems to have an advantage in this case over the forms $8.5792-10$ and $7.4921-10$ as the operation under the first arrangement is performed easily and with little confusion. Again let it be required to subtract $\bar{3}.9241$ from $\bar{1}.7439$. The form of the logarithms may now be changed to good advantage thus:</p>
--	--

$\begin{array}{r} 9.7439-10 \\ 7.9241-10 \\ \hline 1.8198 \end{array}$	<p>Here obviously is a simple case of subtracting one binomial from another, and the result explains itself.</p>
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In the problem $(.039)^5 = ?$ it is necessary to multiply $\bar{2}.5911$ (or $\log .039$) by 5. The result $\bar{8}.9555$ is easily explained. Five times the positive mantissa (or .5911) is $+2.9555$ of which .9555 is the mantissa of the product and $+2$, now a characteristic, is combined with the product (-2) (5) to give -8 . Hence the

$\log .039 = \bar{2}.5911.$
 result is $\bar{8}.9555$. In $\sqrt[5]{.039} = x \quad \log x = \frac{\bar{2}.5911}{5} = \frac{\bar{2}.5911}{3}.$ It

seems that neither the form $\bar{2}.5911$ nor $8.5911-10$ yields readily to division by 3. A brief discussion brings out the fact that the second form would be satisfactory if the minus term of the binomial were divisible by 3. The logarithm is then written $7.5911-9$ and the process is a case of dividing a binomial by a

monomial. Thus $\log x = \frac{7.5911-9}{3} = 2.5303\bar{3} - 3 = 2.5304 - 3 = \bar{1}.5304.$

A multiplication problem like (523) (-472) (-42196) generally causes trouble. As a rule the writer anticipates this by making it clear that the negative signs of the factors are beyond the jurisdiction of logarithms, that the process is performed as if the factors were all positive and that the sign prescribed by the law of signs for multiplication is attached to the final result.

RECITATION 7—COMPOUND INTEREST PROBLEMS
AND EXPONENTIAL EQUATIONS

Two new applications of logarithms are introduced this period—compound interest problems and exponential equations. The formula $A = p(1+r)^n$ is carefully developed and a solution of a specific problem undertaken. As in the former work the proper manner of presenting the solution is strongly emphasized. Following is an example (from a pupil's paper) as to what the requirements are:

Find the amount at compound interest of \$750 for 7 years at 4%.

$A = p(1+r)^n$	$\log 1.04 = 0.0170$	
$\log A = \log p + n \log (1+r)$	$\underline{\quad 7 \quad}$	
$\log A = \log 750 + 7 \log 1.04$	$7 \log 1.04 = 0.1190$	
$\log 750 = 2.8751$		
$7 \log 1.04 = 0.1190$	$.9943 = M\ 987$	$.9941$
$\log A = \underline{2.9941}$	$.9939 = M\ 986$	$.9939$
	$\underline{0004}$	$\underline{0000}$
$A = \$986.50$	$.9941 = M\ 986\frac{1}{4} = M\ 9865$	

The formula $A = p(1+r)^n$ need not be changed for cases where time, principal or rate is to be found. In fact to cause the least confusion in the mind of the pupil any problem involving compound interest should begin with the formula $A = p(1+r)^n$. From this it follows that $\log A = \log p + n \log (1+r)$.

If p is required,

$$\log p = \log A - n \log (1+r).$$

If n is required,

$$n \log (1+r) = \log A - \log p, \text{ and } n = \frac{\log A - \log p}{\log (1+r)}$$

If r is required,

$$n \log (1+r) = \log A - \log p, \text{ and } \log (1+r) = \frac{\log A - \log p}{n}$$

$(1+r)$ is determined
 $\therefore r$ is determined.

Exponential equations are comparatively "easy." It must be emphasized here, however, that in this process a quotient of logarithms is expressly called for and division will not give way to subtraction. An interesting application is provided in the method for finding the log of any number to any base. Required for example to find $\log_9 21$. The solution is easily understood.

$$\text{Let } \log_9 21 = x$$

$$9^x = 21$$

$$x \log 9 = \log 21$$

$$x = \frac{\log 21}{\log 9}$$

In connection with exponential equations the need may arise for division of a logarithm by another logarithm with a negative characteristic. It is well to anticipate it here. In $(.053)^x = .042$

$$x \log .053 = \log .042; \therefore x = \frac{\log .042}{\log .053} = \frac{\bar{2}.6232}{\bar{2}.7243}. \text{ On the sur-}$$

face division seems impossible due to the fact that the denominator is "half" negative and "half" positive, a "house divided against itself." This, however, can be remedied by changing both numerator and denominator to numbers that are entirely negative. Thus:

$$\bar{2}.6232 = -2 + .6232 = -1.3768$$

$$\bar{2}.7243 = -2 + .7243 = -1.2757$$

And the problem is now reduced to division of a negative number by another negative number.

The assignment is made up of problems on compound interest and exponential equations. Two or three members of the class are directed to look up the history of logarithms and prepare short reports on the subject to be read before the class at the next recitation. The remaining minutes of the period are spent on "supervised study."

RECITATION 8—GENERAL REVIEW

The greater portion of this period is devoted to work of a miscellaneous nature. First a few representative problems from those assigned the day before are thoroughly discussed. Then pupils are urged to ask whatever questions they may have relating to the various phases of "Logarithms." Towards the end of the period the reports are read on the history of logarithms, and the class has an opportunity to learn what sacrifices and labors were needed to bring this important and useful branch of Mathematics to its present stage of development.

SUMMARY

The difficulties that the high school pupil encounters in his study of logarithms are of a peculiar nature. The very meaning of the term "logarithm" is not readily grasped. For the mind accustomed to associate an "exponent" with "the number of times a quantity is taken as a factor" will naturally resist any new notion in the premises; and much of this mental inertia must be overcome before the logarithmic relationship can be clearly understood. Nor does the pupil assimilate with ease such unheard-of ideas as a "negative characteristic and a positive mantissa," or "multiplication gives way to addition and raising-to-a-power to multiplication," or "because the characteristic of a logarithm is 3 there will be in the antilog 4 figures before the decimal point." To be rendered digestible these concepts require not a little seasoning and should be allowed to mature in the mind of the pupil. For this and the tremendous amount of drill incident to the study of logarithms we can spare but eight or nine recitations, in some schools spread over a period of three weeks, the length of time intervening between recitations sufficient to erase all such impressions as are not firmly rooted in the minds of the pupils. The teacher must, then, guard against the danger that the pupil's brain be just irritated by a few vague disconnected notions concerning logarithms, these eventually to be lost through disuse in subsequent Algebra.

Yet "Logarithms" is a teachable subject, more so perhaps than a good deal of what is commonly considered "easier" Algebra. Here at last is a subject where all are alike beginners, the in-

ferior pupils, the mediocre, and the brilliant. They bring with them no previous knowledge of the subject, and none is expected of them beyond the bare laws of exponents, and even these are generally taught or reviewed in connection with logarithms. They bring relatively little inferiority complex, born of past failures, for the teacher to combat, few if any erroneous ideas for the teacher to unteach. Here at last is virgin material to begin with and the teacher has an opportunity to lay the foundation and build upon it according to his own designs. Classes in logarithms are generally responsive. The newness of the subject makes the pupils curious; the apparent usefulness of logarithms as a labor-saving device gives them enthusiasm and an eagerness to learn. By unfolding the subject carefully and making his pupil feel at the end of each recitation that they have learned something new and definite the teacher may keep this interest alive to the end. And, when the parts are securely built and assembled, he will find, to his great satisfaction, that his class is in possession of the fundamentals of logarithms and quite skillful in using them.

THE HUMAN SIGNIFICANCE OF MATHEMATICS

By WILLIAM L. SCHAAF
College of The City of New York

What is mathematics? This is a question which it is well nigh impossible to answer in a few concise words. Among other things, the various phases of mathematics endeavor to explain this universe of endless ideas and complex phenomena as an orderly system under the control of external law. Or, as it has been aptly characterized, mathematics is the manifestation of "a passion for infinite harmony in a world of apparent chaos." It is no wonder, then, that Prof. T. P. Nunn regards mathematics as a "glorious intellectual adventure in the expansion of the human spirit," and that its art lies in the noble human effort rather than in the achievement alone. He therefore regards as a basal reason for teaching mathematics, the fact that it represent a line of development of the human mind, a great human and intellectual movement throughout the ages.

Again, somewhere else, Prof. C. J. Keyser expresses the conviction that "hope of improvement in mathematics teaching, whether in secondary schools or in colleges, lies mainly in the possibility of humanizing it." And although he gives no working formula telling how this shall be done, he closes the discussion with this poignant and significant message to teachers: "Knowing the doctrine is essential to living the life."

The present writer, being in complete harmony with the opinions expressed above, as well as with the attitude taken by James Harvey Robinson in his telling little book, "The Humanizing of Knowledge," believes that he can indicate one tangible method of approach to the problem of humanizing the teaching of mathematics. The effort consists, in short, of organizing a system of popular lectures designed to convince people that mathematics has a very vital contact with our daily lives, and that, contrary to popular impressions, it is fraught with human significance.

Several possibilities suggest themselves immediately. Such a course of popular lectures might be organized primarily for the public at large, or for college students, or for teachers in train-

ing, or might even be adapted for presentation to senior high school boys and girls. Each of these appear to be a separate problem, demanding considerable serious study and experimentation.

As a preliminary experiment, the writer has arranged a tentative organization of material and mode of presentation, designed primarily not for any one of the four specific purposes indicated above, but to serve as a source of experience upon which to build further. The remaining portion of this paper is an account of an experimental course which was first given in the fall of 1925 at the College of the City of New York in its evening session.

The general plan and method may be best described by the following letter of transmittal which secured permission from the authorities to give the course.

LETTER OF TRANSMITTAL

Title—The proposed "Special Course" is to be designated: "THE HUMAN SIGNIFICANCE OF MATHEMATICS."

Aim of the Course—The course aims to acquaint the student with:

- (1) the fact that mathematics is a vital, growing, and essentially modern science;
- (2) the simplicity and nature of the fundamental notions underlying mathematics;
- (3) the role of mathematics in modern civilization;
- (4) the human significance of mathematics.

Description and Syllabus—The plan and organization of the course is fully described in the accompanying proposed "Special Folder," which gives the detailed subject matter to be included and the approximate order of presentation.

An additional memorandum summarizes the course briefly, and is suggested as a possible insert in the regular "Bulletin" of the College under the heading of "Unattached Courses."

For Whom Designed—This course is designed to meet the needs of

- (a) such maturer students as are intent upon a liberal education, affording them an insight into the fundamental nature of mathematics;

- (b) such students who have found a natural interest in mathematics and are desirous of obtaining a further conception of what mathematics really is all about, without meeting the requirements of engineering students;
- (c) such interested laymen who wish to acquire a fair understanding of the great underlying ideas of mathematics together with their human worth.
- (d) prospective teachers of mathematics.

Admission—Any mature person interested in mathematics, or anyone seeking, as a part of a liberal education, an insight into the significance of mathematics, may be admitted to the course.

Credits—Such students as are regularly enrolled in any of the Schools of the Evening Session, and who have previously passed Advanced Algebra and either Trigonometry or Solid Geometry, may receive two credits upon successful completion of the course.

Method of Conducting Course—It is proposed to conduct the course two evenings a week for one semester.

The course will consist of approximately twenty formal lectures, the remaining time being devoted to informal discussions and quizzes.

In addition, students taking the work for college credit are required to do specified collateral reading, to submit a definite number of written reports on assigned reading, and a definite number of original essays on selected topics. Lastly, there will be the usual type of final examination.

In view of the non-existence of a suitable single text covering the entire scope of the course, it is proposed to use GRIFFIN's "Introduction to Mathematical Analysis" during the first half of the course as a textbook, to be supplemented by the collateral reading referred to above.

Conclusion—It might be well to emphasize the fact that the course is *not* a course on

"Popularized Calculus,"	"Fundamental Concepts,"
"History of Mathematics,"	"Philosophy of Mathematics,"
or "Applied Mathematical Analysis."	

It is, however, unique in that sufficient orientation in each of the above fields will be undertaken to accomplish the aims of the course as set forth above.

Throughout the course continued emphasis will be placed on fundamental notions such as the function, rate of change, series, variables and limits, infinity; and on their significance in the multifarious fields of human endeavor, including Science, Industry, Practical Arts, Business Ethics, and Philosophy.

The essential simplicity of these concepts will also be stressed.

Inasmuch as the writer was not permitted to give credit for the course, it was deemed desirable to omit the assigned reading, informal discussion, requiring of essays, the final examination, and to abandon the use of the Griffin text. The course therefore shaped itself into a series of more or less formal, though intimate lectures.

It may be of interest to note the announcement of the course as it appeared in the College Bulletin, as follows:

HUMAN SIGNIFICANCE OF MATHEMATICS

(See special folder)

To the generally well-educated person Mathematics all too often appears to be an exceedingly unromantic study devoid of all human interest. The aim of this course is to disclose the essential simplicity and nature of the fundamental notions underlying Mathematics; to show that Mathematics is a vital, growing, and essentially modern art, and to trace the romantic role played by her in the development of modern civilization.

OUTLINE OF THE COURSE

- A. Relatedness in Life and Nature
- B. Growth and Change
- C. The Law of Organic Growth
- D. Choice and Chance
- E. Civilization and Numbers
- F. Infinity and Eternity
- G. The Democracy of Mathematics
- H. Some Human Values.

Any mature person interested in mathematics, or any one seeking as a part of a liberal education an insight into the genuine significance of mathematics, may be admitted to the course, provided the student be familiar with the elements of algebra and geometry.

One term; 1 hour a week; Fee, \$2.50.

The "special folder" referred to above was a circular distributed in advance to assist in securing the necessary publicity, and may be regarded somewhat as a syllabus, summarizing the content of the lectures and the sequence intended. A reprint of this circular follows.

THE HUMAN SIGNIFICANCE OF MATHEMATICS

It is the aim of this course to point out that Mathematics, albeit the most ancient of the Sciences, is nevertheless flourishing today, unsurpassed by any rival. It is proposed to delineate some of the basic notions that underlie the science; to cultivate an appreciation of the power of Mathematics, and to trace the role that Mathematics and abstract thinking in general have played in the development of civilization. And finally, it is hoped to demonstrate that Mathematics is not only intimately concerned with Humanity, but that it is replete with spiritual values and intensely pregnant with human significance.

SYNOPSIS OF THE COURSE

RELATEDNESS IN LIFE AND NATURE

1. *The Function Concept.* The fundamental question of variation; intimate relationships between quantities of every description; the significance of such relationships to the mechanic, merchant, farmer, housewife, scientist, and engineer; the problem of exhibiting such variation and relatedness; graphical methods; distinctive types of variation; periodicity; the concept of the funeton; physical science the study of variation and relatedness in Nature; how the function idea permeates modern science; functional relationships and their mathematical treatment in modern business, industry and economics; mathematical curve and equation.

GROWTH AND CHANGE

2. *The Limit Idea.* The notion of instantaneous speed; average speed in an interval; the necessity of the idea of limiting values; lengths, areas and volumes; changing direction and instantaneous direction; slopes and tangents; more about variables and limits; the limit concept in Science.

3. *The Everchanging.* The mode of measuring a rate of change; the notion of the derivative; differentiation of simple

functions; maxima and minima in Nature; time rates of change; related rates; the rate of a rate of change; velocity and acceleration; "nothing permanent but change" in Nature.

LAW OF ORGANIC GROWTH

4. *Series and Their Consequences.* The properties of progressions; the basic ideas involved in the theory of investment; the accumulation process; the notion of present value; annuities; infinite series and their behavior; convergence and divergence; the ever-recurring limit-concept; limit-processes as ideals, and idealization in thought and human aspirations; the Natural law of civilization as an increasing exponential function of time.

5. *The Power of the Exponent.* The exponential function exceedingly common in Nature; the Compound Interest Law and its intimate connection with natural phenomena; the process of organic growth; die-away curves; Newton's law of cooling; some fundamental laws of chemistry and physics; physiological growth.

CHOICE AND CHANCE

6. *The Laws of Probability.* The background of arrangements and selections; the toss of a penny; the idea of chance; mathematical conception of probability; games of chance and their futility; normal binomial distribution; errors of observation; the probability curve; physical science and probability; theory of measurement; kinetic theory of gases; the law of chance in biology; evolution and heredity.

7. *Modern Business and Probability.* A necessity of modern society; the fundamental principle of many sharing mutually the misfortune of the few; the scientific basis of life assurance; probabilities of life; the law of mortality; expectation of life; life contingencies combined with interest; the mathematical correctness of rate-making; probability and the telephone business.

CIVILIZATION AND NUMBERS

8. *Number Systems and Number Concept.* Primitive counting; origin and evolution of number systems; the quinary and vigesimal systems; the Babylonian sexagesimal system; Hindu notation. The invention of decimals; and the principle of position; their influence on civilization. The role of zero; the introduction of the negative; extension of the number system to include

the fraction and the irrational; further extension of the number system necessary; the imaginary, a recognized analytic instrument; Argand's geometrical interpretation; Gauss and the complex number; scalars and vectors; their significance in the arts and sciences.

INFINITY AND ETERNITY

9. *Mathematical Concept of Infinity.* Dynamic and static aspects of infinity; the concept of infinity as a limit. The notion of classes; what cardinal numbers are; one-one reciprocal correspondence; equivalent classes; whole and part; the distinction between finite and infinite; transfinite classes and cardinals; the denumerable infinity; some denumerably infinite classes; the cardinal of the continuum; the "glorious" linear Continuum.

10. *Philosophy of the Infinite.* Zeno's paradoxes; can Achilles overtake the tortoise?; the flying arrow always at rest; the queer dilemma of Tristram Shandy; the Lucretian conception of infinitude; is the Universe infinite in extent?; the naive answer of Lucretius; the possibility of a finite, yet unbounded realm; Pascal's poetic picture and its mathematical significance; the axiom of infinity.

THE DEMOCRACY OF MATHEMATICS

11. *Foundation of Geometry.* Euclid's elements; Euclidian definition, axioms and postulates; logical difficulties; a new world; an abstract science vs. a concrete application; the logical significance of definitions, axioms and postulates; banishing the self-evident truth; primitives; the notion of a class; the doctrine, or set of assumptions; propositional functions; the democracy of geometry; Russel's humorous dictum; pure mathematics and applied.

12. *Non-Euclidian Geometry.* A Non-Euclidian world; Euclid's attitude toward his parallel postulate; ancient times and Middle Ages; Wallis, Saccheri, and Legendre; Lobatchewsky and Bolyai; the parallel postulate not provable; Rieman's geometry; the consistency of non-Euclidian geometry; the three geometries closely related; which geometry is correct?

13. *What is Mathematics?* Inadequacy of the familiar conception of mathematics as the science of magnitude; the science of indirect measurement; the geometry of position; group concept;

futility of defining mathematics by enumerating its domains or content; the method of Mathesis; distinguishing characteristics of mathematical thought; the "science which draws necessary conclusions;" this apodictic quality due to the nature of the concepts with which it deals; the abstract point of view. The object of mathematical thought—to think the logically thinkable; disparity between mathematics and natural science; the aim of the scientist; Mathematics at once a muse and a willing servant.

SOME HUMAN VALUES

14. *Mathematics and Art.* The intrinsic beauty and harmony of mathematics; number and the world order; the arithmetic of beauty; duality and trinity; symmetry and balance; law of rhythmic diminution; geometry in nature; latent geometry; ornament from mathematics; magic squares and their translation into pattern; geometry and architecture; frozen music; mathematics of sound and color.

15. *Humanity and Mathematics.* The qualities of mathematics ideal for the handling of Life's problems; variable and constant as the counterpart of fixedness and change in human affairs; equation concept as the reflection of natural and moral law in life; mathematical limits and limit processes omnipresent as Ideals and Idealization in all thought and human aspirations; functionality and interdependence in life; infinitude and soul expansion; spiritual significance of the permanence of mathematical laws and the contemplation of ideas under the form of eternity; Mathematics as the abstract idealization of life.

16. *Summary and Examination.* The art of engineering; Korzybski's concept of man; what Time-binding means; human ethics as Time-binding; the role of rigorous thinking in the history of civilization; the human significance of mathematics.

One of the difficulties which presented itself was the uncertainty as to what type of students were going to take the course, and the previous mathematical training of these students. In order to fortify himself against any contingencies on this score, the writer made use of the following questionnaire at the very outset, which aided materially in integrating the mathematical experience of the group before him.

QUESTIONNAIRE 1

- (1) *Name*.....
- (2) *Address*.....
- (3) *Where did you first learn that this course was to be given?*
.....
- (4) *If from the circular, where did you obtain it?*
- (5) *Are you taking any other courses at C. C. N. Y. this semester?*.....
- (6) *Have you ever studied—*
 (a) *Trigonometry*.....
 (b) *Advanced Algebra*.....
 (c) *Other Mathematics (Please indicate)*.....
- (7) *Are you a teacher of Mathematics?*.....
- (8) *Are you contemplating taking advanced courses in Mathematics?*
- (9) *If your neighbor, an ordinary intelligent layman, were to ask you: "What is mathematics, anyway? What is it all about?" What would you tell him?*

As a result of the use of this questionnaire there was found to be a very great divergence in the extent of the students' previous mathematical experience and training. There was among the group a student who had studied differential equations, vector analysis, and mathematical philosophy. From this extreme, their experience ran down to the other extreme of the minimum requirement of a knowledge of the elements of algebra and geometry. Needless to say, this range of differences complicated the lecturer's problem; how to stay on a level not too far above some, and yet not bore others?

As the course proceeded, various minor details in the presentation and organization were modified, as the occasions and needs

of the students dictated. In the main, however, the sequence and content followed rather closely the original plan, with the following exceptions.

In the first place, it was decided not to plunge directly into the matter of variation and functionality without any preliminary treatment; the first lecture, therefore, was devoted entirely to providing an adequate setting, or background. This included a discussion of the development of the scientific movement since the days of Bacon, involving, as it did, the setting aside of the human factor in scientific work; then the rapidly accelerated progress of science to a high degree of specialization, creating as it did, a still further de-humanization of science; and finally, a discussion of the present need for re-humanizing science, and knowledge in general, with all that this implies.

Secondly, it was felt desirable to follow this introduction with the lectures tracing the growth of number systems and artificial numbers, and to develop this topic in slightly greater detail than had been originally intended. This was suggested by the reaction of students.

From that point on the general plan was carried out as scheduled.

In order to estimate in some way the actual accomplishment obtained at the end of the course, the following questionnaire was used with some success:

QUESTIONNAIRE 2

In your opinion, to what extent might a moderate amount of mathematical knowledge or training affect a layman's

	A Great Deal	Consider-ably	A Little	Not At All
(1) <i>appreciation of beauty and harmony?</i>
(2) <i>notions of ideals and human aspirations?</i>
(3) <i>appreciation of the various kinds of growth in the world?</i>
(4) <i>ideas concerning the element of chance in the world?</i>

- (5) *conceptions of eternity and infinitude?*
- (6) *understanding of relationship and the interdependence of things upon each other?*
- (7) *appreciation of man's intellectual freedom?*
- (8) *realization of permanence and change in the universe?*
- (9) *religious views and emotions?*
- (10) *knowledge of the history of civilization and human progress?*
- (11) *appreciation of modern industry's debt to mathematics?*
- (12) *appreciation of man's physical limitations and his control of the forces of nature?*

These questions were answered by the students the first evening of the course and again on the last evening. A summary of the results showed a decided and very considerable shift in the answers from the last two columns to the first two columns.

CONCLUSIONS

From this somewhat crude and preliminary experiment, a number of conclusions can readily be drawn. There were two major difficulties which presented themselves, and which were validated both by frank criticism on the part of the students as well as by self-criticism on the part of the lecturer:

First, the lack of uniformity of the previous mathematical training on the part of the students (already mentioned above);

Second, a lack of unity in transition from one topic to another, due partly to the absence of some unifying central thought.

If this type of popular lectures is to be given to the general public, the former difficulty will always be present, undoubtedly to an even greater extent. If adapted for high school boys and girls, however, this difficulty disappears almost altogether; and if presented to regular enrolled college students, or to teachers in training, this difficulty probably assumes a position of intermediate importance.

The latter difficulty can be partly overcome by reorganizing the form and content of the lectures. However, it automatically disappears very largely for all types of audience except the general public, for then there is always the regular mathematics course which is being pursued simultaneously with this lecture course, and to which the lectures can be definitely tied-up.

Both of these problems are receiving study and will be given very considerable attention in future presentation. In concluding, it might be stated that the writer is also engaged in planning out a more elaborate diagnostic test, to consist of a larger number of more specific items; this by way of preparation for further experimentation when such a popular course will be used with control classes.

As a general outcome, then, the conviction may be stated, that although a great amount of continued experimentation is still necessary, this device apparently offers one mode, at least, of driving in an entering wedge toward securing the much-needed re-humanization of mathematics in particular, and of all knowledge and human activity in general.

A PLEASANT APPROACH TO DEMONSTRATIVE GEOMETRY

E. V. SADLEY,
Minneapolis

The introduction of experimental geometry into our junior high school courses and the willingness to call any study of form in the lower grades by the high sounding name of "intuitive geometry" have done much to pave the way for the more formal treatment of the subject in the senior high schools.

We, who have labored to teach proof by superposition in the first two theorems of Book I, experience a sickening apprehension when we contemplate the brooding pall of doubt which threatens to settle over the class after the first few days.

The pupils have loved the investigation of geometric truths by measurement and experiment. Then to be told that the only property of a point is position and in the next breath to be urged to take a point from where it is and place it where it is not is more than human credulity can be expected to accept, especially in the questioning age of adolescence.

Again last September, we faced the possibility of looking into the half convinced countenances of the sophomores as they repeated parrot-like the proof "Place the triangle ABC upon triangle DEF so that, et cetera" and of meeting the inquiry, "Why do we have to prove that? We have drawn the figures, cut them out and they always fit together if we work carefully enough." And after all, is it not an insult to their intelligence?

It took some courage to cast text book to the winds and to venture upon uncharted seas, just to drift a bit in the world of geometric form and symmetry seen all about us in our daily rounds. But time was pressing. The outline of work for the semester had to be covered. However the gracious attitude of acceptance that pervaded the class after a few days was well worth the effort spent on the appreciative side of geometry in the early stages of its study.

Given a simple lesson in the meaning of central, axial and plane symmetry and equipped with a background of knowledge of geometric form gained from their own experience and by their previous study, the class was turned to search books and magazines for interesting material. The children came with pictures of the homes and work of primitive tribes, of the art and architecture of the Greeks and of the middle ages. They were encouraged to open their eyes to geometric forms in their own homes, in their city, and in the textile designs of clothing and decoration. They analyzed rugs, vases, baskets, houses, and churches in their relation to their symmetry. They brought baskets, blankets, samples of silk and prints. On the way to school, they gathered leaves, seeds, flowers and fruits to be examined for their symmetry and form.

After four or five meetings of the class in which were discussed the different phases of individual interest, a list of topics were suggested upon which to organize a theme for English.

1. Do the savages know their geometry?
2. Symmetry in a colonial house.
3. The Greek temple, a perfect example of symmetry.
4. Geometric form in a Gothic cathedral.
5. A local building analyzed from the standpoint of geometry.
6. My day in a world of mathematics.
7. Beautiful bridges.
8. The geometry of a snowflake.
9. Geometry walked with me through the north woods.
10. Symmetry in the plant, trees and flowers.
11. Symmetry a secret of beauty.

One girl who had visited Europe during the summer chose the topic, "The Geometry of Sulgrave Manor" and took us on a personally conducted tour through the house and grounds of the ancestral home of the Washingtons to observe the many examples of geometric forms and symmetry in the architecture, the furniture, and the landscaping of the gardens. To make the theme the more attractive she illustrated it with beautifully colored postcards of the scenes as she described them.

The articles written upon the designs made up primitive peoples in their bead work, basketry, and homes were made extremely fascinating by colored pictures mounted in the body of the theme.

After a few days' study, all agreed, "if this is what geometry is like, I know I shall like it." Having seen how great a part geometry plays in the beauty of the world if only our eyes are trained to appreciate it, we could then turn to the formal treatment of the subject with good grace.

(The themes were later used as a program in the morning assembly. Slides from the public library added to the interest of the talks.)

ODE IN PRAISE OF MATHEMATICS

Oh, geometry, geometry!
A "solid" study, you'll agree;
And what care I how "plane" it be,
If it be not "plain" to me?

"What's trigonometry about?"
You ask me. Well, you'll soon find out,
If you have to study it some day,
It's about the *limit*, so I'll say!

"What's in a name?" great Shakespeare asked;
But then he ne'er his brain had tasked
With secant, ordinate, abscissa,
Co-logarithm, and mantissa.

And angles!—if your brain's "acute,"
Your work is "right," and sure to suit.
But if you chance to be "obtuse,"
To toil and weep is not much use.

Such vexing problems I meet each day
That I'm sure my hair is turning gray;
Is a girl "extreme" if she isn't "mean?"
Are Napier's tables painted green?

If you plant cube roots in the ground, what grows?
And where has polygon, who knows?
And where does an angle get its degree?
And is it B. A. or Ph. D.?

Are there "missing links" in evolution?
And what is dissolved in Descartes' solution?
If you meet an improper fraction, pray,
Should you coldly turn the other way?

Can you hit Ball's theorem with a bat?
Does a cardinal number wear a red hat?
Could the powers of X make a treaty, pray?
And does heat expand a binomial, say?

I'm not a genius, that is clear,
And I don't believe in "sines," I fear,
And numbers, whatsoe'er they be,
Are all "irrational" to me.

Though the law of tangents I can't endure,
I'm not a Bolshevik, I'm sure;
All the "radicals," if I had my way,
Would be sent to Russia this very day.

If I die before Commencement Day,
Bury me where the sunbeams play;
And write on my tomb this epitaph;

"SHE DIED OF AN INCOMMENSURATE GRAPH."

—Mount St. Joseph College, Dubuque, Iowa

TANGENTS

Tangents are changeable things,
They go as if they had wings
From zero to infinity,
Oh, what a wondrous journey it must be!

For quadrants one and three,
Plus they always have to be,
For quadrants two and four
They go to minus infinity and no more.

ODE TO THE COSINE

Oh, Cosine, thou art near divine
(Co-named sister of regal sine)
In going from nothing to ninety
You change from one to zero quickly.

At zero you are one,
At ninety none.
At thirty a square root of three
O'er two, you'll always be.

At sixty you are one o'er two,
In quadrant one you're positive, too,
In two and three you're negative,
In four again you're positive.

So now, my dear Cosine,
I love you better than the sine,
Because you've done your best
And kept me from an awful test.

GLENN WILLIAMS (High School Greeley, Colo.)

THE SINE

Oh, "a" over "c" means sine to me,
The nicest function that could be,
From zero to one it upward hops,
From one to zero quickly drops,
Plus, plus all the way,
From zero to 180.
Then, alas, it is plus no more,
But downward goes through three and four.
Minus, minus it must be,
From 180 to 360.

JAMES EWING

THE TANGENT

"A" over "b" means tangent to me,
What wondrous sights it must see
As it journeys on to infinity,
Up and up through one and three,
Then it begins to trouble me
By turning toward minus infinity.
But says this mystic line to me,
"Pray tell me now what's hard to see
Concerning the change of signs in me.
Strange things happen in infinity
And plus meets minus there, you see."

MILDRED BURBRIDGE (High School, Greeley, Colo.)

THE TANGENT

How lovely is the tangent,
Far better than the cotangent,
Its life must be full of lots of fun,
Because it can travel from earth to sun.

It goes from zero to infinity
Quite sedate but ideally,
That is from zero to ninety degrees
And everything it always sees.

At quadrant one and three it's positive,
But, alas, at two and four it's negative,
Now a poet I could never be,
But how I love to talk about infinity.

JOSEPHINE WATERHOUSE (High School, Greeley, Colo.)

TRAVELING MATHEMATICS

By MISS MARYLEW STRITZINGER

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Upper Darby, Penna.

Imagine its the eighteenth of June. Come on, go abroad with me. Take only your knowledge of Mathematics which includes your ability to express yourself in real English, if possible in French, Italian and German; then remember your Geometric truths in packing your conventional covering, and be prepared to use your keen observation, honesty, accuracy and pep. For this trip lengthen that proverb: "Speech is Silver, Silence is Golden." but PEP is Platinum.

First use that practical formula $d = rt$ and know just when to arrive at the pier in New York. Here the three classes are separated very much as cattle, and when the pushing and shoving as two variables almost reach their limits, board the French liner—"Paris." Well, some boat! Its geometric shape appeals to us: the decks are parallel planes, if you do not think so, beware of mal-de-mer. Now determine your plane by using the deck-chair, the steward and yourself as the three points. Of course you as well as the fish need food, so follow the shortest path to the dining-room. If you are as lucky as I was, you will eat at the captain's table. Captain Fontaine will entertain you in his quarters and show you the entire boat, that is, the engine-room, the cuisine, and the chart-room. Here are thrills for you. Do you not gasp at the mathematical atmosphere as you realize how your life depends on the accuracy of these mathematical instruments? They direct the boat and check that direction as we solve problems and prove that our solutions are correct. "Know your subject and know that you know it." During the day play golf, deck-tennis, shuffle-board, and other games. At three the children enjoy a Punch and Judy show "Guignon;" Movies are at five and dancing in the evening. The rest of the time seems to be for eating with breakfast at nine, bouillon at eleven, lunch at one, tea at four, and dinner at seven.

Just before you call at Plymouth an instrument is lowered into the water; it will gather sand and by observation, compari-

son, and mathematical computation the exact distance from the shore is discovered. On the Fourth of July let us land on French soil at Havre. Just a bit woozy. We are all right but the world just will not be still, and we are mentally bruised by the unruly landscape. Then just as you have about decided that Newton had made a grave mistake you arrive in Paris. A recent writer has said that Paris reminds him of New Year's Eve and Hallowe'en combined, and that the taxi drivers come together in seven opposite directions even though its a perversion of the compass. Washing the streets will increase our taxi prayers and give us all a thrilling, slippery time. Now we will "hyperbolate," "parabolate," and even make ellipses as we rush through Paris, Versailles, Fontainebleu, and the battle-fields. Last summer there was an International Exposition built on both sides of the Seine River which flows through Paris and is crossed by thirty-seven bridges. One of the first places to visit is the Place de l'Opera which is so beautifully pictured in the "Phantom of the Opera;" then the Eiffel Tower—bulwark of the rigidity of the triangle—one thousand feet high, from which you will get a marvelous view of Paris. The Pantheon, a cyclorama of the Great War is a painting on circular canvas depicting the wonderful heroism which each of the allies displayed in the war. You will wish to shake hands with Roosevelt, thank Wilson with your mind, and applaud General Pershing with beating heart. The Louvre is a mecca for everyone. As you know there are many original paintings, and sculpture here with mathematics as the foundation. The Palais de Versailles is one of the most perfect buildings in France from an artistic and instructive point of view. It was built in its original form under Louis XIII, enlarged by Louis XIV to its present immense size, and inhabited by the French kings up to the Revolution. In 1871 the palace was occupied by the German force and on the eighteenth of January, King William of Prussia, was here proclaimed the Emperor of Germany. Can you not feel the French thrills and the German chills when the Treaty was signed in 1918? Gloria Swanson used Fontainebleu in her picture, "Madame Sans Gene," but to walk through this historic place is thrilling. Barbizon is another interesting place where you will see the fields which Milet used for "The Gleaners" and be able to draw in-

spiration from the home and surroundings of this great man. In Paris be careful of your arithmetic as you shop on the Rue de la Pais, the Fifth Avenue of Paris (and they certainly make you pay). Now let us go down Champ Ellysees, the Riverside Drive of New York, to the Bois and while we have tea I will tell you about the trip I took from Paris last year.

When I was forced to leave the city of life, art and style, I learned that there were three classes of travel according to one's taste for crowding. I journeyed to Geneva with two Japs, a Swiss Miss, a French woman, and my American chum. Here we felt the solidity of the Allies as their colors floated from the buildings. Then the next morning we started on a five-day bus trip through the French Alps. I have climbed Pike's Peak never expecting to return to level ground, and I have been hurled across the peaks in Yellowstone Park, and breathlessly I have come down Mt. Talmapias in California by gravity, but this motor trip from Geneva to Nice cannot be described. Perhaps if you were foolish enough to ride in a Ford on the edge of the roof of the Woolworth building you would approach this dangerous tour with a beautiful view as a limit. We motored five days stopping at nights in Annecy where we dodged the lorgnettes held before many curious eyes, in Grenoble, Barcelonette, and Briancon. These towns are quaint, keeping the customs and costumes of old France. The women seemed to do the hard work in the fields, and it was quite the thing to see them washing clothes on the boards at the troughs chatting aimlessly or fighting over their rights to use the troughs. One characteristic which seemed prevalent was that of inviting the animals and insects of the neighborhood to eat with us and they were so chummy. Many a time I ate my lunch with one hand while I threw H_2O at these undesired guests. At the top of these mountains were usually small huts where Napoleon had been, similar to the thousands in America claimed to have been visited by Washington, and here we loitered to get our nerve to descend, which we did shudderingly for only the most skilled driver could keep us on the narrow path. We arrived in Nice and never did anything seem so hospitable as the "ocean waves" of the Mediterranean Sea. Almost immediately we took a taxi over the Cornish Drive to Monte Carlo—the playground of southern France. A few days

later we stopped in Geneva where Columbus first grasped the idea that the earth is round and opened up such a wide field of applications of Spherical Geometry. Here we took a sleeper for Rome; the berths were perpendicular to the engine while our American berths are parallel.

As we arrived in Rome about Seven-thirty and could not get into our room until ten, we walked down town for breakfast. The only place that was open gave us milk with a little coffee; this and spaghetti forced us to live on fruit while in Italy. You have all seen the Coliseum and the Forum but think of them in the historical setting. While Paris showed us a rushing life, Rome went to the other extreme and exhibited all the antiques of the Ages. Then in Florence our guide "talked us" from one place to another making us rave over the grandeur of the galleries. One section in particular on the Ponte Vecchio Bridge over the Arno River was filled with stories dating back centuries. Then a day's travel in an Italian train to Venice, and as we slid into a gondola we marveled at the scene. These so-called romantic gondoliers were having the best arguments they talked so fast and were so angry that it was not until that night in the moonlight that we could picture: "Santa Lucia Home of Fair Poesy." While here we visited St. Mark's Square, Doge's Palace, the Rialto Markets, and Lido, the famous seaside resort of Venice on the Adriatic. After a short stop in Milan we journeyed to Bern, Interlaken, and Lucerne in Switzerland. Every thing was so clean and while we enjoyed Italy it was a relief to really eat, be cool, and comfortable. Our trip up the Rhine was marred because of rain and then we went over to Brussels. Who does not have a deep regard for Belgium in the stand she took in 1916? Although the Germans lived in the city for sometime nothing was harmed due to an agreement with the major. But the Belgians pointed with derision at "Die Deutsche Bank" which was not completed due to the hurried exit of the Germans. A few days later we followed the crowds to Ostend which seemed to be the Atlantic City of Belgium. The bath-houses were on wheels and porters hitched horses to these houses and drove us into the Channel. After a swim it seemed to be the thing to do to sit in the sand and eat dry shrimps which were sold like peanuts. The next day six of us flew to London. Going up on

the switch-back curve and flying ninety miles an hour many feet above the ground were not unlike the trip through the Alps, in fact after a little while we were reading. However the trip down convinced me that hereafter I will discover heights by numerical Trigonometry. Then dusty old London bro't us back to life; this largest city in the world reminded us of New York with an ancient pedigree. As in Paris we enjoyed the theatres, the shopping and the historical places; in fact London held us fascinated and we were loathe to leave for Southhampton where we boarded the "George Washington" of the United States Line.

And now as you have almost finished your tea I will give you a brief summary of certain characteristics. First each large city has a marvelous cathedral: Notre Dame with its wonderful geometric rose windows in Paris, Saint Peters, the largest in the world in Rome, St. Marks with the Lion in Venice, the lovely one in Cologne, and the exquisite one in Milan which I liked the best; the second largest—St. Pauls in London and the renowned Westminster Abbey. Secondly the large cities were built on rivers. The Seine crossed by thirty-seven bridges in Paris, the Arno with its Ponte Vecchio filled with treasures, and the Amerigo Vespucci, and the Thames in London. In France we realized that Napoleon was the greatest man that had ever lived but his reputation suffered in Italy especially in Milan when our guide informed us how Napoleon had ordered lime put on that marvelous example of extreme and mean ratio: "The Last Supper" which Leonard de Vinci painted in the church of Gracie. In France I felt the people were so sad even the children and yet they seemed to be living on their nervous energy; Italy was similar though more bitter toward the Germans who were so extravagant as they traveled in northern Italy. Switzerland was quite neutral and had no animosity toward either faction. Thirdly the theatres were different. The ushers sold you the programs and insisted on tips for service; during intermission dancing was enjoyed in the lobby in Paris but in London tea was served. And lastly one word about the monetary value. In France the franc which was nineteen cents before the war is less than four cents in our money. The Metric System is a joy. What a joy to divide a franc into one hundred centimes; to

know kilometers were on the basis of ten; that there was only one quart to consider: the liter. This system was a delight all through the continent, but in London twelve pence a shilling, twenty shillings a pound and so on. . . In Italy the lira was less than a franc but it is higher again, the Swiss franc is eighteen cents, the English shilling is about a quarter and the Germans have a new mark which is about a quarter also. We found living in Paris less expensive than in America if you shun the large hotels and restaurants, but in London expenses seemed to be higher. And now let us see the value of a franc as we taxi from the Bois to our hotel.

NOTE ON THE FALLACY

By WALTER H. CARNAHAN
Shortridge High School, Indianapolis

There are few phases of mathematics in which high school pupils show more interest than those mathematical curios which we call fallacies. It is because they excite spontaneous interest that fallacies are so often used in mathematics club programs as well as in classes. We speak of them as curios, and that they are. But they may be made to serve at least two useful purposes.

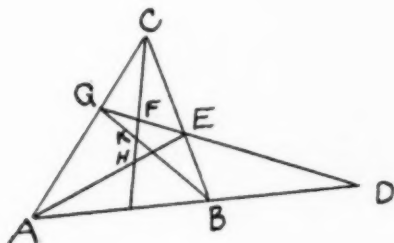
In algebra the student is taught to avoid division by zero. At once he raises the question, "Why?" For one thing, division by zero leads to that absurdity known as the fallacy. Again, he is taught to make careful construction of figures used in geometric demonstrations. Again he asks, "What is the use?" Once more, the answer is that an absurdity or fallacy may result from failure to do so.

Many common mathematical fallacies result from division by zero, or from failure to make accurate construction of figures used in geometric demonstrations. I believe this statement should be the starting point in any club or class study of fallacies. It makes the fallacy serve a useful purpose rather than giving the student the impression that mathematics is not an exact and reliable science.

Little could be gained from reprinting here all the fallacies available for club or class use. The reader interested in these is referred to Ball's "Mathematical Recreations," Jones' "Mathematical Wrinkles" and other similar works. The fallacy given below is here published for the first time. The writer came upon it a few years ago while working out a solution to the theorem of Serentus.

FALLACY

Part of a line segment equals the whole.



Given: Triangle ABC with DG any line through any point on the base extended intersecting the sides of ABC in E and G . F is so located that $DE/DG = EF/FG$. CF , AE and BG are drawn, AE and BG intersecting CF in H and K respectively.

To prove: CK equals FK .

Proof: Considering G as a point on the extended base of triangle BKC we have: (By the lemma of Menelaus: If the three sides of a triangle be cut by a straight line the product of three segments which have no common extremity equals the product of the other three.)

$$(1). \quad GF = \frac{GE \times KF \times BC}{CK \times BE}$$

Considering E as a point on the extended base of triangle ACH :

$$(2). \quad EF = \frac{EG \times AC \times FH}{CH \times AG}$$

Regarding D as a point on the extended base of triangle ABC :

$$(3). \quad \frac{DE}{DG} = \frac{BE \times AC}{BC \times AG}$$

By hypothesis,

$$\frac{DE}{DG} = \frac{EF}{FG}$$

Substituting for the terms of this proportion their equals from (1), (2) and (3) we have:

$$\frac{EG \times FH \times BE \times AC \times CK}{CH \times AG \times EG \times FK \times BC} = \frac{BE \times AC}{BC \times AG}$$

Whence,

$$\frac{FH \times CK}{CH \times FK} = 1; \text{ or, } \frac{FH}{FK} = \frac{CH}{CK}$$

From the figure,

$$FH = FK + KH, \text{ and } CH = CK + KH$$

Therefore,

$$\begin{aligned} \frac{FK + KH}{FK} &= \frac{CK + KH}{CK} \\ \frac{FK}{FK} + \frac{KH}{FK} &= \frac{CK}{CK} + \frac{KH}{CK} \end{aligned}$$

Therefore,

$$\frac{KH}{FK} = \frac{KH}{CK}$$

Therefore, $FK = CK$, which is obviously impossible. Q.E.D.

WHEN IS A PROOF NOT A PROOF.

By P. STROUP
Cleveland, Ohio

We have all heard of pupils who "got by" in geometry by merely memorizing the proofs given in the book and of teachers who permitted them to "get by." To such persons those proofs were not proofs. Assuming that such methods are passé if they have not passed all together, what limits are we to approach in the other direction? How long should a student be asked to hold in his memory any completed proof? Is it sufficient to remember the fact and that it has been proved? Are the proofs presented really designed to convince the student of the truth of the fact being proved? Are they the kind of proof that is really convincing to him? Is the logical proof convincing to him? Why prove to him facts that he is sure are true? We ask why he thinks that two triangles must be alike if they have two sides and the included angle the same and he replies that anybody can see that with the implication that somebody around is a "boob." If he can see no exceptions then is it not a proof to him? Is it not more likely to prejudice him against the geometric proof than to raise any respect for it to drag him through a formal proof under those conditions?

When a student is assigned a completely proved proposition in the book to study for the lesson next day, what is to be expected of him in class? What the student gets or tries to get depends on what he expects to be required of him. What effect is the study of the proof supposed to have on the student? It should convince him of the truth of the fact and be an example of how such a fact can be proved. One will not happen without the other. How should he work to get these results? If his object is mere repetition he will not get them. Suppose he expects the teachers to ask him if he has any questions and he knew that credit would be given for good questions. The teacher need not be the one to answer them. See if he can raise questions that the others cannot answer. Suppose he expects such questions as, why is that proved?, how else might that line be drawn?,

where does this given fact come in? Suppose no repetition of the proof were asked for at all. The student should dissect the proof rather than pack it away in his memory. What is the plan or outline of the proof? Geometries are now in print in which the plan of the proof is given before the details of the proof are begun. Whether this should be done for all proofs is a question, but the student's attention should be called to the mental economy of getting that and keeping it in mind.

It is a fact that at least three-fourths of the propositions that are proved in ordinary plane geometry can be done as originals and indeed it is the exceptional proposition that cannot have its proof worked out by the students with the guidance of a few questions. In most classes there is too much book and not enough teacher. If teachers were what they ought to be and superintendents had confidence that they were, our books would only contain a glossary, a few specimen proofs, and a nicely graduated collection of problems (including theorems to be proved). Why should the teacher be so largely a tester and an explainer of the book? If she knows the subject let her teach it. Every text in geometry is written as if for students that were to have no teacher. The type of text, of course, varies with the subject, but in algebra and geometry we would raise a much higher quality of mathematicians if the course were more of an expedition of discovery through the subject rather than through a certain book. Why? "It says so in the book." Perhaps the same type of mind would reply "The teacher said so" but the proof given orally by the teacher or worked out in the class by the students or presented by a student does not offer the chance for verbatim memorizing that a written proof does.

All the pressure in memorizing should be put on the facts that have been proved. If the students are going to acquire skill in finding proofs they must have a rapid acting set of associations. Equal lines or angles must raise immediately the ideas, congruent triangles, parallels, parallelograms, etc., that they may be sorted over to see which applies. This is where memory and drill must play their important role. The minds of many geometry students are jammed with the memories of proofs where the proof should have faded into sub-consciousness leaving the fact

standing out in clear relief. How often when the student is showing how far he got with an original proof he comes to the place where we ask, "Didn't you think of that?" No, he did not. In the stress of the struggle that association did not come through. It was not strong enough. We do not mean to imply that drill on the facts will make all the students able to get all the originals but it would help to have more facts connected as surely as the isosceles triangle is connected with equal sides and equal angles. Time spent on memorizing proofs would yield better returns if spent on drill on these associations so that they become less forced leaving more power for the general conduct of the proof. More care should be taken to make the proofs convincing to the people to whom they are addressed than in memorizing. A convincing proof takes little effort to memorize, but a memorized proof is not necessarily convincing.

A proof should seldom be memorized for the mere sake of remembering it. The remembering should rather be the by-product of understanding it. This is not true of very long proofs but for the beginner who has not developed a confidence in logic a proof is not convincing, if at all, until he gets far enough away from the details to see the thing as one idea. He will admit that if the triangles are alike the parts must be alike and that if the parts are not alike the triangles cannot be alike, but he loses the general idea while he is working out the details. An algebraic proof if of any length is no more convincing than a slight of hand performance. Their recollections of it next day give one the same impression as if they had had a rabbit pulled out of a hat and they were given the hat and asked to produce the rabbit. In plane geometry then there is little place for memorizing proofs. The best test of the mastery of the details of a proof is the investigation of its converses and opposites. In geometry or out of it it is the thinking around a subject that makes you feel at home with it, turning it over in your mind, looking at it from different points of view and correlating your impressions.

Two of the most futile pages ever written in a geometry for beginners were the two proving that only one perpendicular can be drawn from a point to a line and that it is the shortest dis-

tance, by extending the perpendicular and drawing an oblique on each side. In the first place, no one doubts it. In the second place, the proof is so long that it carries no conviction to the beginner. As a sample of geometric proof it is useless to him. He won't meet another like it until he has entirely forgotten it. Perhaps I am a bit vague as to where intuitional geometry ends and demonstrative geometry begins. The demarcation for a beginner should be vague. The transition should not be abrupt. A sharp drawing of the line is the work for advanced students or teachers. Can anything be a proof in demonstrative geometry that is not stated in the conventional way? If by moving a ruler around the perimeter of a triangle I get you to admit that the sum of the angles is half a revolution, and I do the same for all the triangles that you suggest, are you not compelled to admit that as far as you can see that is true for all triangles? That is more than a laboratory test. That is a general proof. Is it a geometric proof? It is. It is not favored in geometries because it cannot be put in the conventional form. Its early introduction by the teacher would do away with the awkwardness of the proof of several other facts. If you think of a car going around in a circle as turning through the exterior angles of a many sided polygon then you are compelled to admit that their sum must be 360°. Is that a geometric proof? What sanction is it that stamps a thing as a mathematical proof? Does it help the beginner to be assured that all the experts consider this as a proof? Can't he reasonably ask to be shown something that convinces him? Why is it any more necessary to prove to the beginner that only one perpendicular can be drawn from a point to a line than to prove that only one straight line can be drawn between two given points? Is the fact that the former has a geometric proof sufficient reason for inflicting it on the beginner? One rule for logical thinking is to have as few premises as possible. But this reduction of the number of premises to a minimum is hardly the proper sphere for a beginner. He is shown by carefully selected instances that his eye can be deceived and yet you admit that in most cases things are as they appear. Instead of learning where the eye is likely to be deceived he is to distrust all appearances. Some students are so depressed in the matter

of distrusting their judgment from visual experience that they lose a valuable source of suggestion in forming geometric theories. We had the problem of how many pipes can form a circular bundle. One boy thought it improper to experiment to get a working hypothesis. We need more confidence in the natural truth of the subject. The truth will out, not in the form laid down by the mature mind highly trained in the subject, but with fumbling and feeling and temporary inconsistency. When our time comes to go floating over the Polar regions on a week-end cruise we will not get anything like the thrill of the occupants of the Norge. Can't we save for our students a little more of the thrill of pioneering even if we have to sacrifice the precision of schedule and operation of a perfect passenger service?

A proof is a means of transferring from one mind to another an increased feeling of the certainty of a statement. But an idea is not identical for any two minds even when the fact is as abstract and colorless as two plus two is four. What is a proof to me may not be a proof to you. A statement is made and after a moment's thought I admit that it is true but you with a larger experience bring out an exception to the statement. Now as an occasional experience that is all right and I only suffer a slight shock to my confidence, but how disastrous if that is repeated many times. This is what must happen if a student tries to get the meaning in many of the proofs offered. Instead of the proofs meeting any skepticism that is in the student's mind, the skepticism is first set up, in full view of the student, he is showed how to bowl it over and then he is asked to repeat the words. It is a sham battle. It is a sham without a battle. It is child's play, setting up a lot of straw men and then after the gun is all aimed they are permitted to pull the trigger and admire the way they fall. If the geometric proof were presented not for itself, but for the sake of proving something to somebody who was not convinced it would have a much better chance to show off.

The facts of geometry are all abstractions. The figures in geometry are all ideals. An abstraction is a mental bundle of concrete experiences. Now when we consider the size of these

bundles in the teacher's mind and in the student's mind we get **some** idea of how weak this thing that we call a complete proof is to the student. If he looks over his stock of ideas for exceptions to the statements and he finds those ideas vague and indefinite, how can that proof mean much to him? Can there be two perpendiculars to the same line at the same point? The response is not the ringing "no" that we hoped to hear. The indirect proof is not convincing to beginning students for this reason. Absurdity or impossibility is probably strong language for the way it strikes the students. They really show their independence of mind more by their uncertainty than they would by an echo-like response. Their experience gives them very narrow foundations for their beliefs.

It is generally assumed that knowledge of geometry and faith in its facts is to be built like a monument. First the foundation is laid and having been laid it is not to be changed nor can the plan of the building be changed. It is rather built as a city grows. The main street remains, widened and lined with higher and deeper buildings, rivaled and perhaps outclassed by other thoroughfares, the far out districts become populated and joined to the main street by once crooked paths now straightened and paved, the old is torn down and takes its place in the larger new. It is very wasteful and if enough foresight were possible and enough capital available much of the waste might be avoided for the city, but is there any other way to develop for a mind and still be the mind of an individual or at least a happy, useful and productive one in this world. Experience, experience, experience, from experience alone comes conviction. We have faith in logic because we have tested it by experience, not simply because it is logic. We expect a logical proof to carry the same conviction to a student as it does to us. It cannot. He is not capable of putting the statements against the background of fact that makes them stand out so clearly for us. We can only expect that he check the statements against what experience he has, keep an open mind, and try to be consistent.

Some one has said that the syllogism is not the form of thinking. It is the form in which to put the results of our thinking. The same may be said of the formal proof. It is not the form in

which the proof evolves from our thinking but rather a conventional form in which a proof can be put after it has been born with all the accompanying travail. So if it is said that all these formal proofs are presented as examples for the students to follow, one answer is that what students need most is ideas and not packing cases for them. Do they need so many examples? Do those geometries which are so largely a series of formal proofs accomplish much toward the end of making the student sure of the facts and helping him to test the truth of his own ideas? Asking the student to think in the form of the proofs given in the book is like asking a beginning student in a language to think in that language. It is more reasonable to ask, let, implore, incite, inspire, urge, him to think in any language and then if possible get him to translate the idea into the formal language of geometry as it is in the book.

I have tried to show some probability of the truth of the following propositions:

That a logical proof in the conventional form may not be convincing to the student.

That a mathematical proof can be put in other than the conventional form.

That the attempt to make the beginning propositions rest on rock bottom is more confusing than convincing to a beginner.

That the development of the student should be given preference over the perfect development of the subject.

That the teacher be permitted to teach the subject rather than explain a book on the subject.

That repetition is not as good a test of mastery of a proof as the ability to apply its details in the proof of the converses, opposites, and other related propositions.

That the pressure in memory and drill should be put on the formation of associations such as equal arcs, equal chords, equal angles.

(Both visual and vocal associations should be used).

NEWS NOTES

Frequently the Editor is asked for information concerning standing offers of prizes for the solutions of mathematical problems. For example, one reader wishes to know whether there is an offer of a prize for the solution of the problem of trisecting an angle with straightedge and compass. This particular reader is convinced that he has a solution of the problem and wishes information concerning any prizes that may now be available to one who solves the problem. The Editor should appreciate information concerning any standing offers of prizes in any way relating to the solution of problems in mathematics.

(J. R. C.)

The Democrat and Chronicle of Rochester, New York, has published a monograph consisting of reprints of a series of articles describing a program of instruction pursued in the public schools of Rochester, New York, published in the *Democrat and Chronicle* during the year 1926. Twenty-six of these concern the elementary school, thirty-seven of these deal with the Junior high school and twenty-one deal with the Senior high school.

The annual meeting of the Association of Teachers of Mathematics of the Middle States and Maryland was held in Buffalo, November 26. Professor Wilfred H. Sherk of the University of Buffalo read a paper on "Honor Courses in Mathematics." Mr. Robert S. Binkerd, Vice-Chairman of the Committee on Public Relations of the Eastern Railroads, read a paper entitled, "The Part Mathematics Has Played in the Development of the Railroad." The program was prepared by the officers, Dr. H. Ross Smith, President, Mr. Clarence V. Scoborio, Treasurer, and Miss Elsie V. Bull of West Chester, Pa.

The subscription price of the *American Mathematical Monthly*, the Official Journal of the Mathematical Association of America,

is Five dollars per year to non-members of the Association, and Four dollars to members.

Raleigh Schorling, University of Michigan, was one of the speakers at the October meeting of the West Virginia State Teachers' Association.

Mr. C. M. Austin, of Oak Park, Illinois, is presenting the work of the National Council of Teachers of Mathematics to numerous Teachers Associations.

W. D. Reeve of Teachers College, Miss Marie Gugle, President of the National Council of Teachers of Mathematics and Mr. C. B. Marquand of Columbus, Ohio, addressed the Mathematics Section of the Central Ohio Teachers Association.

Dr. John R. Clark of the Lincoln School, read a paper on the Need of Changes in Geometry Teaching at the Albany Section of the New York State Teachers Association and at the New Haven and Bridgeport sections of the Connecticut State Teachers Association.

The Curriculum Revision Committee of the city of Denver has recently published a Course of Study Monograph for mathematics in grades 7, 8, and 9. The monograph is the result of the work of teachers and specialists extending over a very considerable amount of time.

The regular Fall meeting of the Philadelphia Section of Mathematics Teachers of the Middle States and Maryland was held Thursday, October 21, 1926, at 6 o'clock, in Kugler's Restaurant. Sixty-eight members and guests were present.

After the dinner a brief business meeting was held, Mr. Brecht, the president, presiding. It was voted to waive the reading of the minutes of the Spring meeting.

The resignation of Mr. J. Albert Blackburn, vice-president, was accepted. The nominating committee, appointed by the

president, consisting of Mr. Moore, Chairman, Miss Morin, and Miss Shollenberger, presented the name of Mr. Harry M. Shoemaker, of North East High School, to succeed Mr. Blackburn. It was moved, seconded and carried that the report of the nominating committee be accepted, and Mr. Shoemaker was declared elected vice-president.

The president called the attention of the members to the first Yearbook of the National Council.

Mr. Brecht then introduced the speaker of the evening, Dean Raymond Walters of Swarthmore College. Dean Walter's topic was "How to Improve the Relation between the High Schools and Colleges."

Mr. M. C. Boyd of the Music department of the University of Pennsylvania, added to the evening's pleasure by rendering two piano solos.

After a vote of thanks to Dean Walters, the meeting was adjourned.

Walter Beyer, a pupil in Milton Academy at the completion of his course in plane geometry, was able to prove that equilateral triangles ACD , ABA and BCE are constructed upon the sides of AC , AB , and BC in triangle ABC , the lines DB , CF , and AE are concurrent. Have you any geometry pupils who can prove this? Walter Beyer's proof will be published in the February number of the Teacher.

MEMBERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

(Continued from November issue)

PENNSYLVANIA

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(To be continued)

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OFFICIAL BALLOT

FOR PRESIDENT—(One to be voted for)

- ☐ Marie Gule, Columbus, Ohio
☐ C. A. Austin, Venice, California

FOR VICE-PRESIDENT—(One to be voted for)

- ☐ C. M. Austin, Oak Hill, Ill.
☐ C. N. Stokes, Minneapolis

FOR SECRETARY-TREASURER—(One to be voted for)

- ☐ J. A. Foberg, State Dept. of Public Instruction, Penna.
☐ E. W. Schreiber, Maywood, Ill.

FOR MEMBERS OF EXECUTIVE COMMITTEE—(Two to be voted for)

- ☐ Vera Sanford, Lincoln School, New York
☐ H. C. Christofferson, Normal School, Oskosh, Wis.
☐ Wm. Betz, Rochester, New York
☐ T. J. Johnson, Normal College, Chicago, Ill.

NOTICE: Mark your ballot and mail it *before* February 1, 1927, to Mr. J. A. Foberg, Secretary, Camp Hill, Penna.

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